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Theoretical Basis of Isochronal and Modified Isochronal Back-Pressure Testing of Gas Wells

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ABSTRACT

The theory of gas well testing is briefly reviewed and the theoretical validity of the isochronal and the modified isochronal tests is demonstrated for conditions of laminar flow.

INTRODUCTION

THE isochronal and modified isochronal methods of gas well testing are in common use these days. Both of these methods could be called unsteady-state methods, as the reservoir is not required to reach steady state during the course of these tests. The practical utility of these tests is well established, but there seems to be no completely satisfactory discussion of their theoretical basis. The manual of gas well testing published by the Oil and Gas Conservation Board of Alberta(1) gives a theoretical explanation for the isochronal test using the radius-of-drainage concept. For the modified isochronal test, the analysis presented in the Board manual is incomplete. In this short paper, theoretical bases for both tests are developed from the unsteady-state theory.

Review of Basic Theory

The problem of the radial unsteady-state laminar flow of a gas in a porous medium may be approximated by

$$\frac{\partial^2 P_t}{\partial r_d^2} + \frac{1}{r_d}\frac{\partial P_t}{\partial r_d} = \frac{\partial P_t}{\partial t_d} \qquad (1)$$

where all variables are in dimensionless form,

$$P_t = \frac{(P_f^2 - P_s^2)}{P_f^2}\frac{1}{m} = \text{dimensionless pressure}$$

$$m = \frac{Q}{Q^*}\frac{Q\mu_a T_a Z_a}{.709 \times 10^{-6} K h P_f^2} = \text{dimensionless flow rate}$$

$$r_d = \frac{r}{r_s} = \text{dimensionless radius}$$

$$t_d = \frac{2.634 \times 10^{-4}\, K P_a t}{\mu_a \phi\, r_s^2} = \frac{t}{t^*} = \text{dimensionless time}$$

The factors Q^* and t^* are assumed to be constants in making the flow equation dimensionless. This implies that μ_a, T_a, Z_a and P_a must be constant over the duration of the flow test. Clearly, there will be some variations in these factors for the flow period of interest. The variations are, however, small for the flow periods during which a back-pressure test is conducted, and the over-all effect of these changes is negligible. The solutions to *equation (1)* for various boundary conditions have been obtained. The case of constant production rate is of interest here and has been summarized by Aziz and Flock(2). The dimensionless pressure at the sandface is given by

$$P_t = \frac{1}{2}(\ln t_d + 0.8097) \qquad (2)$$

Equation (2) applies if

$$100 \leq t_d \leq t_{de} \qquad (3)$$

where t_{de} is the dimensionless time before which the reservoir behaves like an infinite reservoir. Note that a complete solution of the boundary-value problems associated with *equation (1)* yields P_t as a function of time and position in the reservoir. Here, only the pressure at the sandface is of interest. Hence, P_t in *equation (2)* and in the following development refers to the sandface pressure, which is only a function of dimensionless time.

The equation defining dimensionless pressure may also be written as

$$\frac{P_f^2 - P_s^2}{P_f^2} = mP_t \qquad (4)$$

In the case of multiple constant flow rates, *equation (4)* may be modified to(3)

$$\frac{P_f^2 - P_s^2}{P_f^2} = m_1 P_{t1} + (m_2 - m_1) P_{t2} + (m_3 - m_2) P_{t3} + \ldots + (m_n - m_{n-1}) P_{tn} \qquad (5)$$

where t_{di} is the total time elapsed since the commencement of the ith flow rate, P_{ti} is the dimensionless pressure evaluated at dimensionless time t_{di}, m_i is the ith dimensionless flow rate and $i = 1, 2, \ldots, n$.

BACK-PRESSURE TESTING

The objective of a back-pressure test is to obtain a relationship of the form

$$Q = c(P_f^2 - P_s^2)^n \qquad (6)$$

for steady-state flow in the reservoir. The use of this relationship is well known and will not be discussed here. It is not always possible or desirable to measure the sandface pressure, P_s, (or the related well-head pressure) for various flow rates, Q, under steady-state conditions, and techniques have been devised which allow the prediction of the relationship of *equation 6* from unsteady-state testing.

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Isochronal Testing

In this type of test, a well is produced at a constant flow rate for a fixed period of time and then the well is shut in to allow the build up of pressure to a constant value (close to original shut-in pressure). This procedure is repeated for other flow rates, with production each time being for the same period. *Figure 1* shows schematically the usual type of variation of P_s with time obtained in the isochronal testing of gas wells. The values of P_f^2-P_s^2 thus obtained are plotted against Q on log-log paper. It is assumed that the slope of the P_f^2-P_s^2 vs Q plot for an isochronal test will be the same as that of the corresponding steady-state curve. Thus, from one steady-state measurement and the slope of the isochronal curve, the steady-state relationship of the form of *equation (6)* may be constructed. Under ideal and laminar flow conditions, the slope of the steady-state back pressure curve is unity. It is usually assumed that any deviations from unity in the slope, due to turbulence or other factors, will be the same regardless of the flow period chosen for the isochronal test. The departure from unity of the slope of a steady-state back-pressure line may be attributed to (1) turbulence or other factors which make the Darcy's law inapplicable, (2) skin effect or other reservoir inhomogenities and (3) two- or three-dimensional flow effects. The over-all effect of such factors is usually small, and it seems reasonable that the slope of an isochronal test would be approximately equal to the slope of a steady-state test. Note that for a very large flow period, the isochronal test becomes a conventional steady-state back-pressure test. If the assumptions stated above are justified, then all that is needed is to show that under ideal and laminar flow conditions the theoretical slope of the plot of an isochronal test is unity. This may be done by substituting *equation (2)* in *equation (4)* to obtain

$$P_f^2 - P_s^2 = \tfrac{1}{2}(\ln t_d + .8097)P_f^2 m$$

$$= \tfrac{1}{2}(\ln t_d + .8097)\ \frac{P_f^2}{Q^*}\,Q$$

$$\text{or } P_f^2 - P_s^2 = CQ \qquad (7)$$

where $C = \tfrac{1}{2}(\ln t_d + .8097)\ \frac{P_f^2}{Q^*}$

is a constant, because t_d, P_f and Q^* are all constants in isochronal testing.

Equation (7) will therefore yield a line of unity slope when plotted on log-log coordinates.

Modified Isochronal Testing

This type of test is similar to the isochronal test except that the flowing time and the shut-in time after each flow rate are deliberately made the same. *Figure 2* shows qualitatively the pressure-time trace for this type of test. Again, to demonstrate the theoretical validity of the test, it is necessary to show that a plot of P_f^2-P_s^2 vs Q on log-log coordinates will yield a line of unity slope. This may easily be done by making use of *equation 5*. To keep the treatment simple, consider a test where a well is produced for a time Δt at a flow rate Q_1, at the end of which the sandface pressure is P_{s1}. The well is then shut in for the same time Δt and then produced again at a flow rate Q_3 for the time Δt. At the end of the second flow period, the pressure at the sandface is P_{s3}. It is desired to show that

$$\frac{\log (P_f^2 - P_{s3}^2) - \log (P_f^2 - P_{s1}^2)}{\log Q_3 - \log Q_1} = 1 \qquad (8)$$

which is the same as

$$\frac{\log (P_f^2 - P_{s3}^2) - \log (P_f^2 - P_{s1}^2)}{\log \frac{Q^3}{Q^*} - \log \frac{Q_1}{Q^*}} = 1$$

or

$$\frac{P_f^2 - P_{s3}^2}{P_f^2 - P_{s1}^2} = \frac{m_3}{m_1} \qquad (9)$$

Equation (9) may be verified by the use of the theory of unsteady-state gas flow.

From equations *(2)* and *(5)*, it follows that

$$\frac{P_f^2 - P_{s1}^2}{P_f^2} = m_1(\tfrac{1}{2}(\ln\Delta t_d + .8097)) \qquad (10)$$

$$\frac{P_f^2 - P_{s3}^2}{P_f^2} = \frac{m_1}{2}(\ln 3\Delta t_d + .8097) + \left(\frac{0-m_1}{2}\right)(\ln 2\Delta t_d + .8097) + \left(\frac{m_3 - 0}{2}\right)(\ln\Delta t_d + .8097) \qquad (11)$$

Equation (11) may be simplified to

$$\frac{P_f^2 - P_{s3}^2}{P_f^2} = \frac{m_1}{2}(\ln 3\Delta t_d - \ln 2\Delta t_d) + \frac{m_3}{2}(\ln\Delta t_d + .8097)$$

$$= \frac{m_1}{2}\ln(3/2) + \frac{m_3}{2}(\ln\Delta t_d + .8097) \qquad (12)$$

Combining *equations (10)* and *(12)*,

$$\frac{P_f^2 - P_{s3}^2}{P_f^2 - P_{s1}^2} = \frac{m_3}{m_1} + \frac{\ln(3/2)}{\ln\Delta t_d + .8097} \qquad (13)$$

It may be seen that the plot of log (P_f^2-P_s^2) vs log Q will yield a line of unity slope for a modified isochronal test provided that the last term on the right-hand side in *equation (13)* is much smaller than $\frac{m_3}{m_1}$; i.e.

$$\frac{m_3}{m_1} >> \frac{\ln (3/2)}{(\ln\Delta t_d + .8097)} \qquad (14)$$

Similarly, for the third flow rate (m_5) the following may be derived

$$\frac{P_f^2 - P_{s5}^2}{P_f^2 - P_{s1}^2} = \frac{m_5}{m_1} + \frac{m_1 \ln (5/4) + m_3 \ln (3/2)}{m_1 (\ln\Delta t_d + .8097)}$$

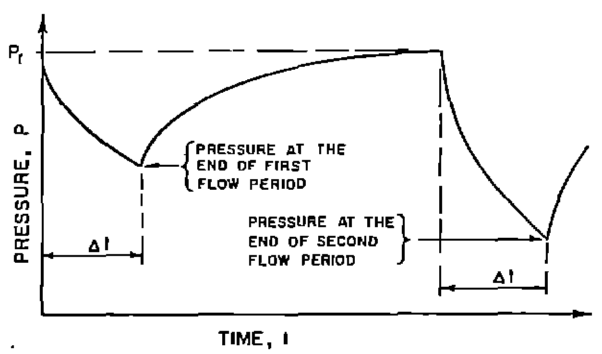

Figure 1.—Variation of sandface pressure with time in isochronal testing.

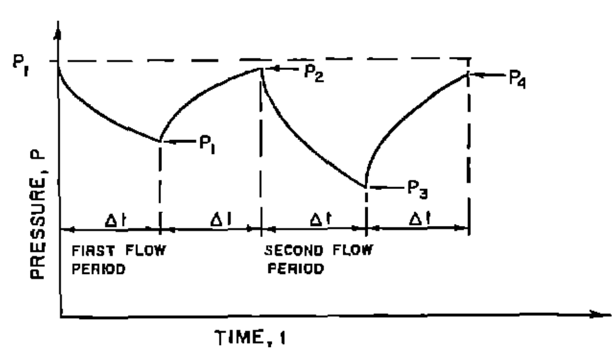

Figure 2.—Variation of sandface pressure with time in modified isochronal testing.

Again, for the test to be valid

$$\frac{m_5}{m_1} >> \left(\frac{m_1 \ln(5/4) + m_3 \ln(3/2)}{m_1 \ln(\Delta t_d + .8097)}\right) \qquad (15)$$

In most practical cases, Δt_d is of the order of 10^5 and conditions *(14)* and *(15)* are satisfied (1).

Another Modification of the Isochronal Test

The test is conducted as shown in *Figure 2*, but the results are plotted in a different manner (1). In this type of a test, $P^2_f - P^2_{s1}$, $P^2_{s2} - P^2_{s3}$, $P^2_{s4} - P^2_{s5}$, etc., are plotted against m_1, m_3, m_5, etc., respectively. This test may also be analyzed by the method used for the previous modification of the isochronal test. It may be easily shown that for this test

$$\frac{P^2_{s2} - P^2_{s3}}{P_f^2 - P_{s1}^2} = \frac{m_1}{m_3} + \frac{\ln(3/4)}{\ln \Delta t_d + .8097} \qquad (16)$$

and

$$\frac{P^2_{s4} - P^2_{s5}}{P_f^2 - P_{s1}^2} = \frac{m_5}{m_1} + \frac{m_1 \ln(15/16) + m_3 \ln(3/4)}{m_1 \ln(\Delta t_d + .8097)} \qquad (17)$$

It is interesting to note that the modification of the isochronal test presented in this section deviates by a smaller amount from a line of unity slope than the test of the previous section. It should also be noted that the deviation is of the opposite sign in the two cases.

Conclusions

It has been shown that under ideal and laminar flow conditions both the isochronal and modified isochronal tests are theoretically valid as long as the dimension time is larger than 100 but smaller than the time at which the influence of the reservoir boundaries is felt. For reasons stated earlier, this conclusion should also be applicable under actual reservoir conditions where, due to "turbulence" and other factors, the slope of the steady-state test line deviates somewhat from unity.

Acknowledgment

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References

(1) "Theory and Practice of the Testing of Gas Wells," *Oil and Gas Conservation Board*, Calgary, Alberta (1965).
(2) *Aziz, K.*, and *D. L. Flock*, "Unsteady-State Gas Flow—Use of Drawdown Data in the Prediction of Gas Well Behaviour," C.I.M. *Bulletin*, 56, 131 (1963).
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Nomenclature

- h = pay thickness in feet
- K = permeability in millidarcys
- m = dimensionless gas flow rate
- P_a = average pressure in psia
- P_f = shut-in formation pressure in psia
- P_s = flowing sandface pressure in psia
- P_D = dimensionless sandface pressure
- Q = gas flow rate in millions of cubic feet per day at 60°F and 14.65 psia
- r = radial distance from the center of well in feet
- r_d = dimensionless radius
- r_w = well radius in feet
- T_a = average temperature in °R
- t_d = dimensionless time
- Z_a = average compressibility
- μ_a = average viscosity in centipoise
- ϕ = porosity

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